Aggregation fallacies in the theory of the firm

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Abstract: The theory of profit-maximizing firms and competitive markets suffers from fatal aggregation errors. The welfare comparison of monopoly and perfect competition is definitive only under restrictive conditions. The accepted profit-maximization formula is in general false: except for monopoly, profit is maximised where own-output marginal revenue exceeds marginal cost. The theory of the firm should be based on empirical research on the behaviour of actual corporations, which finds that marginal revenue and marginal cost are irrelevant.
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The theory of the firm reaches a number of well-known strong conclusions about the price and output decisions and welfare implications of different market structures:

- Perfectly competitive industries produce an aggregate quantity at which the market price in equilibrium equals the marginal cost of production, whereas a monopoly produces a lower quantity at which marginal cost equals marginal revenue. The level of output is lower and the price higher under monopoly than under perfect competition;
- Perfect competition results in the maximization of consumer surplus, whereas monopoly results in a transfer of some consumer surplus to the producer as well as a deadweight welfare loss;
- From Stigler's Relation (George J. Stigler, 1957), profit-maximizing behavior converges to the perfectly competitive ideal as the number of firms in the industry rises. For example with 50 firms and a market elasticity of demand of -2, the market price will be within one per cent of the competitive ideal.

Figure 1 (taken from N. Gregory Mankiw, 2004) puts the first two standard arguments that perfect competition is welfare efficient while monopoly involves a deadweight loss of welfare, a lower level of output, and a higher price.

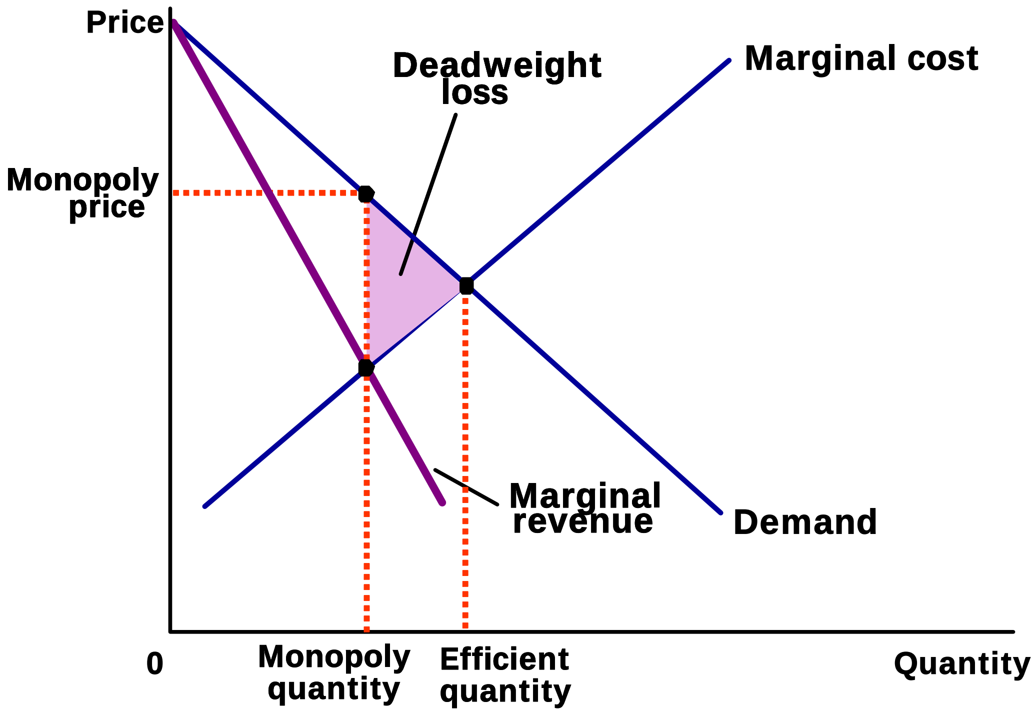

Figure 1: Standard welfare comparison of monopoly and perfect competition

Though these results are accepted as general in economics, they in fact depend upon hitherto unrecognised aggregation conditions. Proper consideration of these conditions invalidates all three popularly accepted conclusions.

1 Marginal cost equivalence

The welfare comparison of competitive and monopoly market structures depends on one obvious aggregation condition: that the marginal cost curve for the monopoly is identical to the supply curve of the competitive industry. If these curves differ, then as Figure 2 illustrates, definitive welfare comparisons cannot be made because it is quite possible for a monopoly to produce a larger quantity at a lower price because of lower marginal costs (Joseph Alois Schumpeter and Redvers Opie, 1934; Paula G. Rosput, 1993). A monopoly would then produce a higher output at a lower cost, and generate a greater level of consumer welfare, even though it was subject to a deadweight loss while the competitive market was not. While this condition is obvious, it has not been recognized that it also imposes a condition upon the total product curves from which marginal product and hence marginal cost are derived.

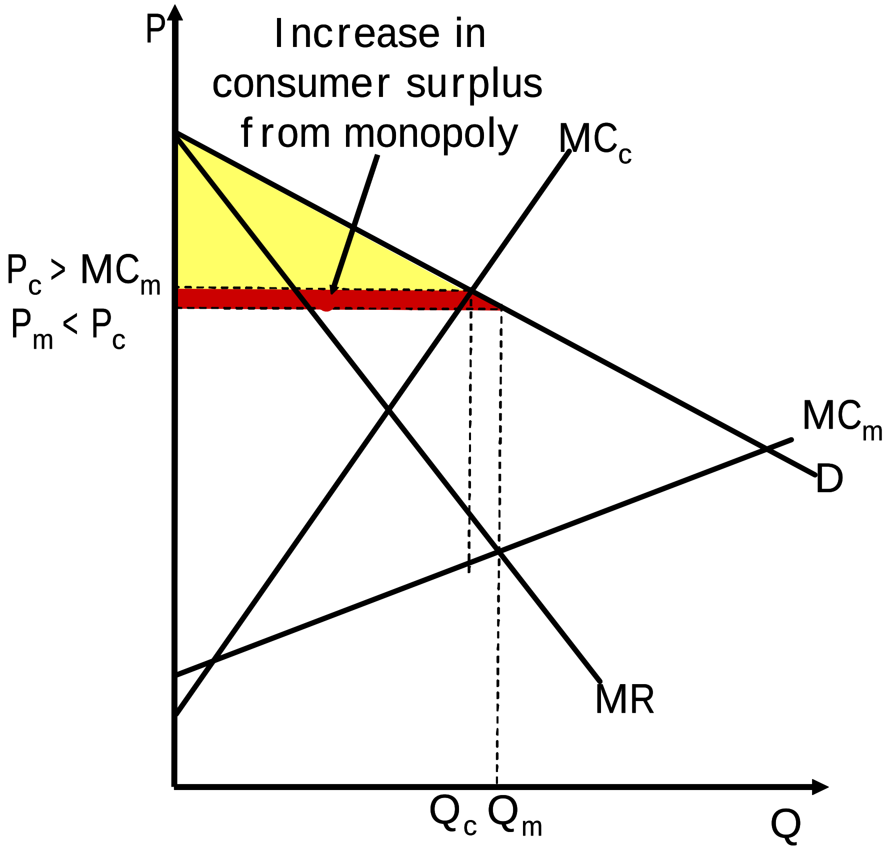

Figure 2: Higher consumer welfare with monopoly if marginal costs differ

The condition that the marginal cost function of the monopoly and the supply function for the competitive industry are identical is simultaneously the condition that marginal products are identical—since a difference in marginal products is the only theoretically allowable source of a difference in marginal cost. If marginal products are identical, then the integrals, total products, can only differ by a constant. If we take labor as our variable factor of production, then with zero output at zero labor input, the constant of integration can be set to zero.

We can put this condition into the following form: given the same number of variable inputs, the production function of the monopoly must be identical to the sum of the production functions of the competitive firms. If we consider (without loss of generality) n identical competitive firms each having one plant employing x workers, and a monopoly with m identical plants each employing y workers, then the output of n competitive firms must equal the output of the monopoly, where $n \cdot x \equiv m \cdot y$ (it would be expected that $n>m$ and $y>x$). Figure 3 puts this graphically: for the welfare comparison of a competitive industry to a monopoly to be definitive, their production functions must coincide for all input levels:

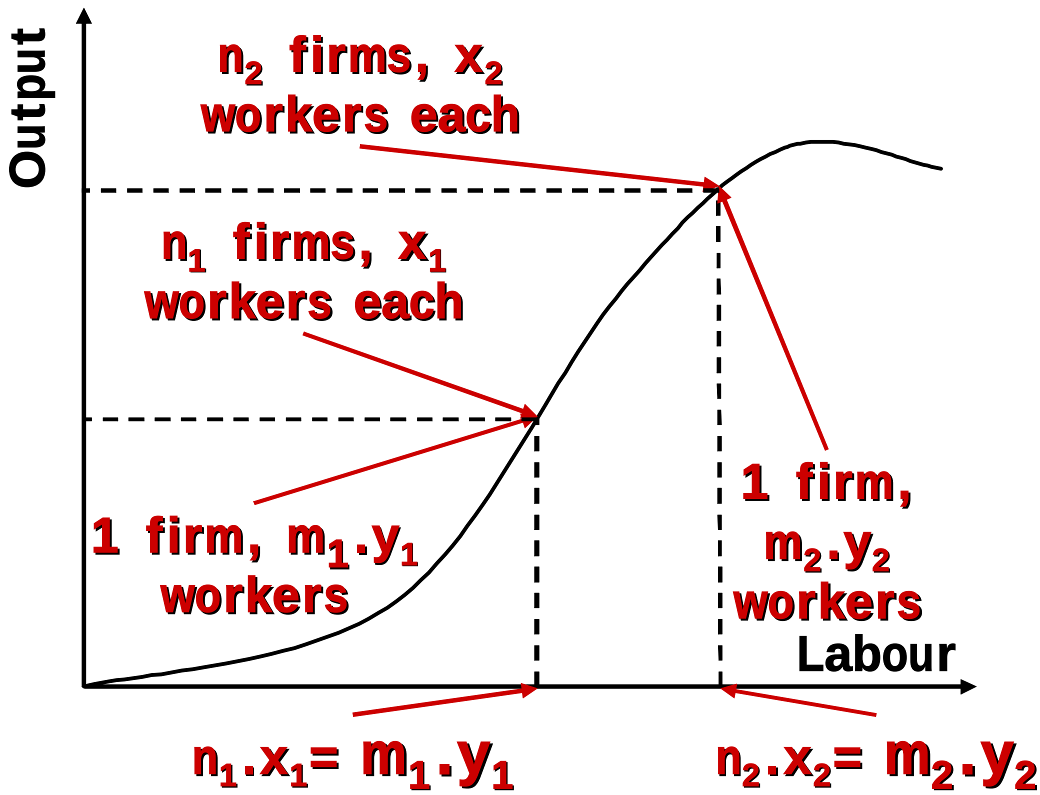

Figure 3: Identical marginal costs require identical production functions

It is easily shown that this condition is fulfilled in two and only two cases. Either:

- The competitive firms and the monopoly have constant identical marginal costs; or
- The monopoly is formed solely by taking over and continuing to operate *all* the plants of the competitive firms.

Using *f* for the production function of the competitive firms, and *g* for the production function of the monopoly, the condition is:

(1) $n \cdot f(x) = m \cdot g(y)$

Substitute $y = \frac{n \cdot x}{m}$ into (1) and differentiate both sides by *n*:

(2) $f(x) = \frac{x}{m} \cdot g'(\frac{n \cdot x}{m})$

This gives us a second expression for *f*. Equating these two definitions yields:

(3)
$$\frac{g(\frac{n \cdot x}{m})}{n} = \frac{x}{m} \cdot g'\left(\frac{n \cdot x}{m}\right)$$
or
$$\frac{g'(\frac{n \cdot x}{m})}{g(\frac{n \cdot x}{m})} = \frac{m}{n \cdot x}$$

The substitution of $y = \frac{n \cdot x}{m}$ yields an expression involving the differential of the log of *g*:

(4) $\frac{g'(y)}{g(y)} = \frac{1}{y}$

Integrating both sides yields:

(5) $\ln(g(y)) = \ln(y) + c$

Thus *g* is a constant returns production function:

(6) $g(y) = C \cdot y$

It follows from (6) that *f* is the *same* constant returns production function:

(7) $f(x) = \frac{m}{n} \cdot C \cdot \frac{n \cdot x}{m}$

With both f and g being identical constant returns production functions, it follows that the marginal products and hence the marginal costs of the competitive industry and monopoly are constant and identical. The general rule, therefore, is that welfare comparisons of perfect competition and monopoly are only definitive when the competitive firms and the monopoly operate under conditions of constant identical marginal cost.

The one exception to this occurs where $n=m$ and therefore $x=y$, in which case the condition collapses to $f(x)=g(x)$, which can be fulfilled by any production function—including one displaying diminishing marginal productivity. In general however, diminishing marginal productivity is incompatible with a definitive comparison of the welfare effects of monopoly and perfect competition. If either the monopoly or the competitive industry is subject to rising marginal cost, then (apart from the exceptional case, which we consider further below) these curves cannot coincide and no definitive welfare comparison can be made.

Figure 2 therefore illustrates the general rule of differing marginal costs—and one feasible outcome that a monopoly could produce a higher output than a competitive industry given sufficiently lower marginal costs. The accepted welfare outcome (Figure 1) must therefore be revised to be shown as the exceptional of two special cases, where the monopoly comes into being by taking over and operating all the firms of the competitive market. In the more general special case where the monopoly operates on a different scale to the competitive firms (so that $n \neq m$ and $x \neq y$) but with identical constant marginal cost, the welfare comparison is as illustrated by Figure 4.

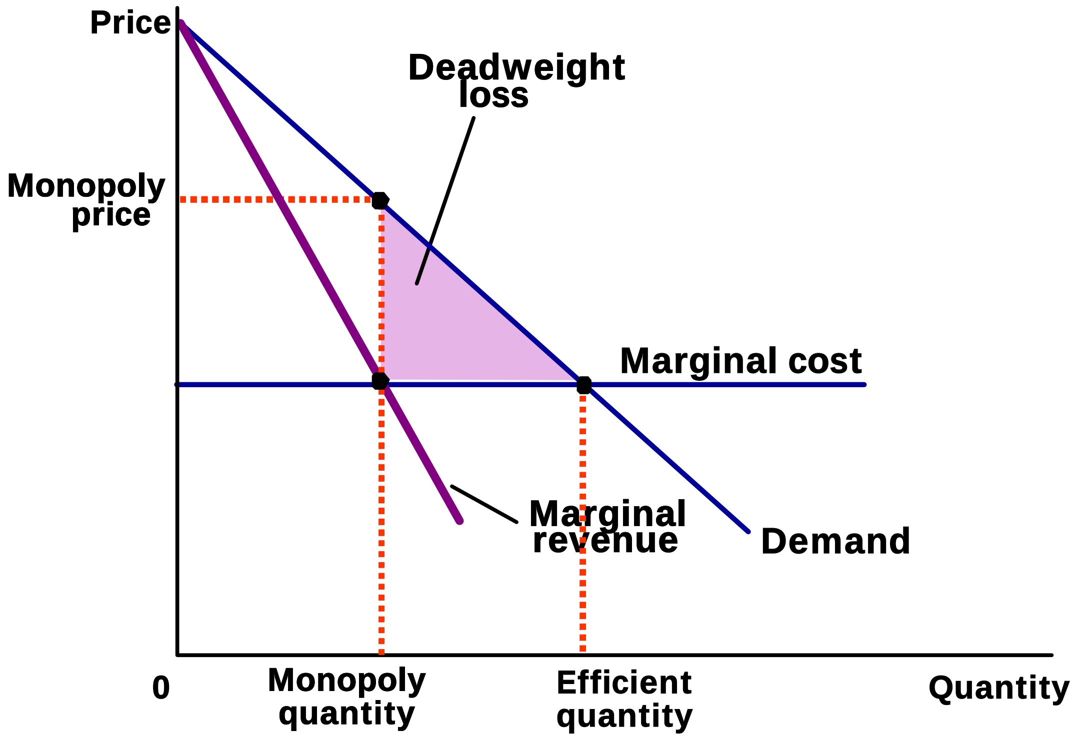

Figure 4: Welfare comparison, special welfare-comparable case

This diagram contradicts undergraduate instruction in economics, where the difference in behavior between a monopoly and a competitive firm is justified on the grounds that the demand curve facing the individual competitive firm is horizontal.[2] This argument is unsustainable in the case of constant identical marginal costs, since the output of the competitive firms would be indeterminate.

It has of course been known since Stigler (1957) that this proposition is false: the slope of the demand curve facing the individual competitive firm is not horizontal but is instead identical to the slope of the market demand curve. Stigler's proof of this made succinct use of the Chain Rule:

(8) $\frac{dP}{dq_i} = \frac{dP}{dQ}\frac{dQ}{dq_i} = \frac{dP}{dQ}$

The relation $\frac{dQ}{dq_i} = 1$ is a simple consequence of the proposition that firms are independent:

(9) $$\frac{dQ}{dq_i} = \frac{d}{dq_i}\sum_{j=1}^{n} q_j = \frac{d}{dq_i}q_i$$

Stigler proposed a reformulation of marginal revenue for the individual firm (assuming n identical firms) which showed that the individual firm's marginal revenue converged to price as the number of firms in an industry increased:

(10) $$\frac{d}{dq_i}(P \cdot q_i) = P + q\frac{dP}{dQ} = P + \frac{Q}{n}\frac{P}{P}\frac{dP}{dQ} = P + \frac{P}{n \cdot E}$$

Stigler argued that this meant that marginal revenue for the i^{th} firm converged to market price as the number of firms increased—"this last term goes to zero as the number of sellers increases indefinitely" (Stigler 1957: 8). We return to this later. At this stage we will use Stigler's Identity $\frac{dP}{dq_i} = \frac{dP}{dQ}$ to establish the second aggregation fallacy: equating marginal cost and marginal revenue is not a profit-maximizing strategy in a multi-firm industry. We then consider the implications of this in three cases:

- The special welfare-comparable case: $n \cdot f(x) = m \cdot g(y), n < m, x > y$
- The exceptional welfare comparable case: $n \cdot f(x) = m \cdot g(y), n = m, x = y$
- The general non-welfare-comparable case: $n \cdot f(x) \neq m \cdot g(y)$

2 Corrected profit maximization formula

The accepted profit maximization formula is derived from the proposition that the point of equivalence of marginal revenue and marginal cost identifies the quantity that maximizes the gap between total revenue and total cost. This proposition is only strictly true for a monopoly. In a multi-firm industry, the i^{th} firm's total revenue is a function not only of its own behavior, but also the behavior of all the other firms in the industry:

(11) $TR_i = TR_i\left(\sum_{j \neq i}^{n} q_j, q_i\right)$

Defining Q_R as the output of the rest of the industry ($Q_R = \sum_{j \neq i}^{n} q_j$), a change in revenue for the i^{th} firm is properly defined as:

(12)[3] $dTR_i(Q_R, q_i) = \left(\frac{\partial}{\partial Q_R}P(Q) \cdot q_i\right)dQ_R + \left(\frac{\partial}{\partial q_i}P(Q) \cdot q_i\right)dq_i$

The accepted formula ignores both the effect of the first term on the firm's profit. It is therefore obvious that the quantities produced will not be profit-maximizing if all firms apply the standard formula.

However it is possible to work out a general profit-maximization formula for the single firm by first establishing the aggregate industry output level that would result if each firm in the industry did equate its marginal cost to its own-output marginal revenue. In this derivation we continue with Stigler's Identity $\frac{dP}{dq_i} = \frac{dP}{dQ}$, and the special welfare-comparable case of constant identical marginal costs $MC(q_i) = MC$.

$$
\begin{aligned}
\sum_{i=1}^{n}\left(\frac{d}{dq_i}(P(Q)\times q_i - TC_i(q_i))\right) &= \sum_{i=1}^{n}\left(P(Q)+q_i\frac{d}{dq_i}P(Q)\right)-\sum_{i=1}^{n}\left(\frac{d}{dq_i}TC_i(q_i)\right)\\
&= nP(Q)+\sum_{i=1}^{n}\left(q_i\frac{d}{dQ}P(Q)\right)-\sum_{i=1}^{n}MC\\
&= nP(Q)+\frac{d}{dQ}P(Q)\sum_{i=1}^{n}q_i - n\cdot MC\\
&= nP(Q)+Q\frac{d}{dQ}P(Q)-n\cdot MC\\
&= (n-1)P(Q)+\left(P(Q)+Q\frac{d}{dQ}P\right)-n\cdot MC\\
&= (n-1)P(Q)+MR(Q)-n\cdot MC\\
&= 0
\end{aligned}
\tag{13}
$$

Equation (12) can be rearranged to yield:

(14) $MR(Q)-MC=-(n-1)(P(Q)-MC)$

Since *n-1* exceeds 1 in all industry structures except monopoly, and (leaving perfect competition aside for the moment) price exceeds marginal cost, the RHS of (13) is negative. Thus industry marginal cost *exceeds* marginal revenue if each firm equates its own-output marginal revenue to marginal cost, so that part of industry output is produced at a loss. These losses at the aggregate level must be born by firms within the industry, so that firms that equate their own-output marginal revenue to marginal cost are producing part of their output at a loss.[4]

Equation (13) can be used to derive the actual profit-maximizing quantity for the industry and for the i^{th} firm within it:

(15) $\sum_{i=1}^{n}\left\{MR_i(q_i)-MC-\frac{n-1}{n}\cdot(P(Q)-MC)\right\}=MR(Q)-MC$

Setting this to zero identifies both the industry-level output Q_K that maximizes profits, and the individual profit-maximizing strategy:

(16) $MR_i(q_i)-MC=\frac{n-1}{n}\cdot(P(Q_K)-MC)$

This formula obviously corresponds to the accepted formula for a monopoly. However for a multi-firm industry, (16) indicates that firms maximize profits, not by equating their own-output marginal revenue and marginal cost, but by producing where their own-output marginal revenue *exceeds* their marginal cost.

This aggregation error-corrected profit-maximization formula results in industry output levels that are in general independent of the number of firms, and equal to that predicted for a monopoly by the aggregation error-affected accepted formula. We illustrate this outcome by considering the two cases in which welfare comparisons can be made: firstly the special case of *n*-firms with identical constant marginal costs, and secondly the exceptional case where the monopoly is formed simply by taking over all the competitive firms.

Special welfare-comparable case: $n\cdot m\equiv x\cdot y, n\neq m, x\neq y, n\cdot f(x)=m\cdot g(y), f(x)=g(x)=C\cdot x$

With a linear demand curve $P(Q)=a-b\cdot Q$ and *n* firms facing constant identical marginal costs *MC*, marginal revenue for the i^{th} such firm producing output q_i is $MR_i=P-b\cdot q_i$. Assuming identical firms, equation (16) indicates that the profit maximizing level of output per firm is the solution to:

$$
\begin{aligned}
P-bq-MC&=\frac{n-1}{n}(P-MC)\\
q&=\frac{1}{2}\frac{a-MC}{n\cdot b}
\end{aligned}
\tag{17}
$$

With *n* firms in the industry, total output is $\frac{1}{2}\frac{a-MC}{b}$, which is independent of *n* and identical to the level the accepted but erroneous formula gives for a monopoly (the one case in which it is correct). The accepted formula predicts the much higher output level per firm of $q = \frac{a-MC}{(n+1)\cdot b}$, and aggregate output that is a function of n: $Q = \frac{n}{n+1}\frac{a-MC}{b}$.

A numerical example indicates how substantially the accepted formula deviates from actual profit maximization. Given $P(Q) = 100 - 10^{-9} \cdot Q$, $n=1{,}000{,}000$ and $MC=\$25$, the accepted formula recommends a per-firm output level of 75,000 units, which is twice the correct profit-maximizing level of 37,500 units. The individual firm's revenues are $1,875,004, $468,746 lower than the profit-maximizing level of $2,343,750; profits are $5.62 per firm versus $1,406,250; and the output produced above the profit-maximizing level is being sold at an average per unit loss of $12.50.

Equation (13) can also be used to derive a formula for "all-industry marginal revenue" for the i^{th} firm (as opposed to "own-output marginal revenue") that is consistent with aggregate marginal revenue:

(18)[5] $$\sum_{i=1}^{n}\left(MR_i(q_i) - \frac{n-1}{n}P(Q)\right) = MR(Q)$$

Own-output marginal revenue for the i^{th} firm using the correct profit-maximizing formula is $62.4999625 compared to a market price of $62.50, and is $37.50 more than marginal cost. However aggregate industry marginal revenue is $25, equal to marginal cost. The summation term for the i^{th} firm in (17) goes to zero as $n \to \infty$, so that the firm sees no revenue advantage in increasing its output beyond this level. If firms follow the accepted formula, the summation term for the i^{th} firm is negative and industry marginal revenue is -$50.

Therefore in the special welfare-comparable case, both competitive firms in the aggregate and monopoly produce the quantity at which aggregate marginal revenue equals marginal cost. Both market structures have identical welfare implications, and the deadweight loss that has been previously attributed solely to monopoly behavior is in fact due to profit maximizing behavior. Figure 5 indicates the accurate aggregate welfare comparison in the special welfare-comparable case of constant identical marginal costs.

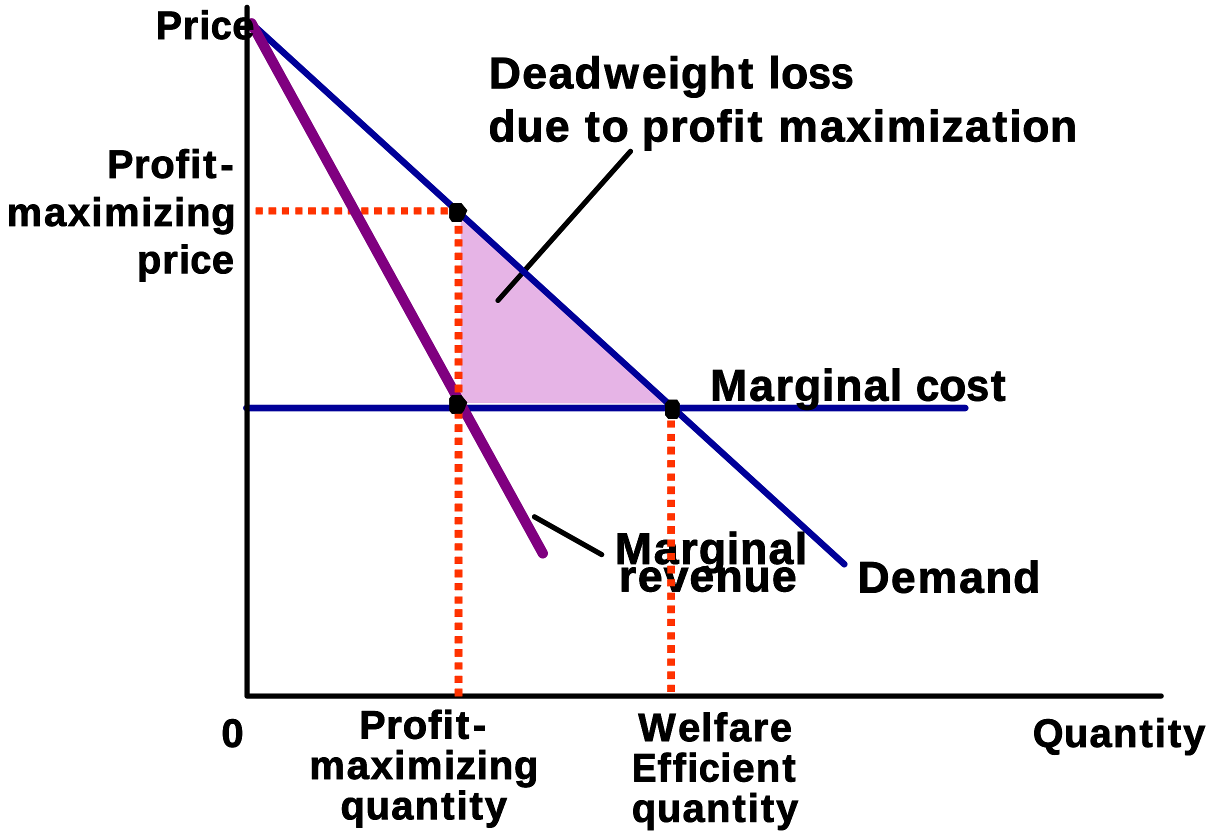

Figure 5: Output and welfare, special welfare comparable case

Exceptional welfare comparable case: $n = m, x = y, n \cdot f(x) = m \cdot g(y), f(x) \equiv g(x)$

In this situation, an aggregate marginal cost curve can be derived by horizontally summing individual firm/plant marginal cost curves. Figure 5 shows the case of identical linear rising marginal cost curves for two firms where, for each firm, $mc(q) = d \cdot q$[6]

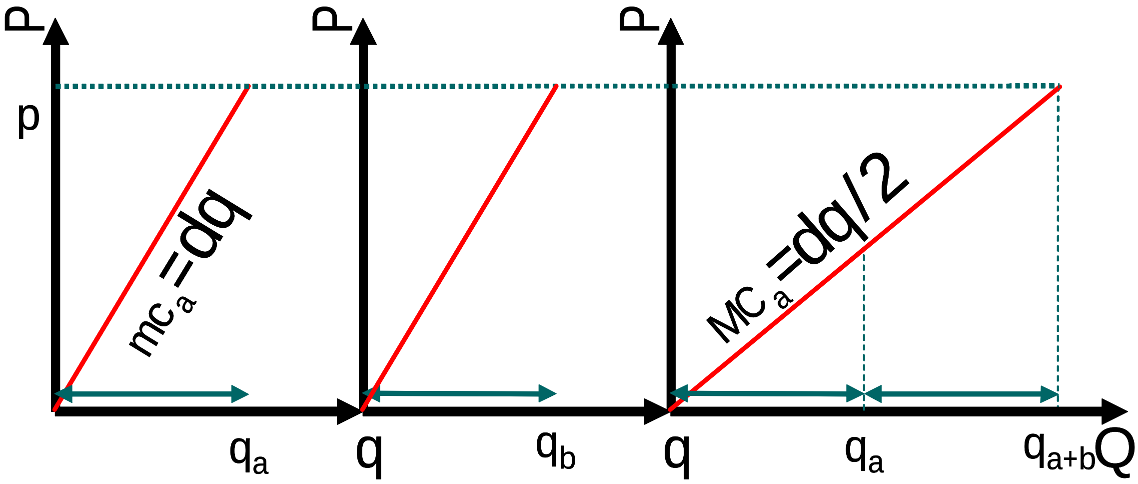

Figure 6: Marginal cost aggregation with identical rising marginal costs

The aggregate marginal cost curve has the same integral as the sum of the individual curves (thus satisfying our aggregation condition for the exceptional case of rising marginal costs and identical scales of operation).[7] It also has the property that $MC(Q) = \frac{d}{n} Q$ where as before n is the number of firms in the industry. This in turn means that $MC(Q) = mc(q)$

(19) $\frac{d}{n} Q = \frac{d}{n} \cdot n \cdot q$

We make use of this relation in the derivations below.

The accepted profit-maximization formula directs competitive firms to equate marginal cost and marginal revenue. This results in the following output levels for the individual firm (q) and the industry (Q):

(20)
$$\begin{aligned} mr(q) - mc(q) &= 0 \\ (a - b \cdot Q - b \cdot q) - d \cdot q &= 0 \\ q &= \frac{a}{(n+1) \cdot b + d} \\ Q &= n \cdot \frac{a}{(n+1) \cdot b + d} \end{aligned}$$

The accepted formula advises the monopoly to equate industry-level marginal cost and marginal revenue, and therefore operate each of its n plants at the level q. This results in the clearly lower industry output level Q:

(21)
$$\begin{aligned} MR(Q) - MC(Q) &= 0 \\ a - 2 \cdot b \cdot Q - d \cdot q &= 0 \\ q &= \frac{a}{2 \cdot n \cdot b + d} \\ Q &= n \cdot \frac{a}{2 \cdot n \cdot b + d} \end{aligned}$$

Our aggregation error amended formula advises the competitive firms to maximize profit by setting the gap between their own-output marginal revenue and marginal cost to *(n-1)/n* times the gap between market price and market marginal revenue:

$$\begin{aligned} mr(q) - mc(q) &= \frac{n-1}{n} \cdot (P - MC(Q)) \\ (a - b \cdot Q - b \cdot q) - d \cdot q &= \frac{n-1}{n} \cdot (a - b \cdot Q - d \cdot q) \\ q &= \frac{a}{2 \cdot n \cdot b + d} \\ Q &= n \cdot \frac{a}{2 \cdot n \cdot b + d} \end{aligned} \tag{22}$$

This accurate profit-maximization rule results in the competitive firms producing at the same level as the monopoly under conditions of rising identical marginal cost.

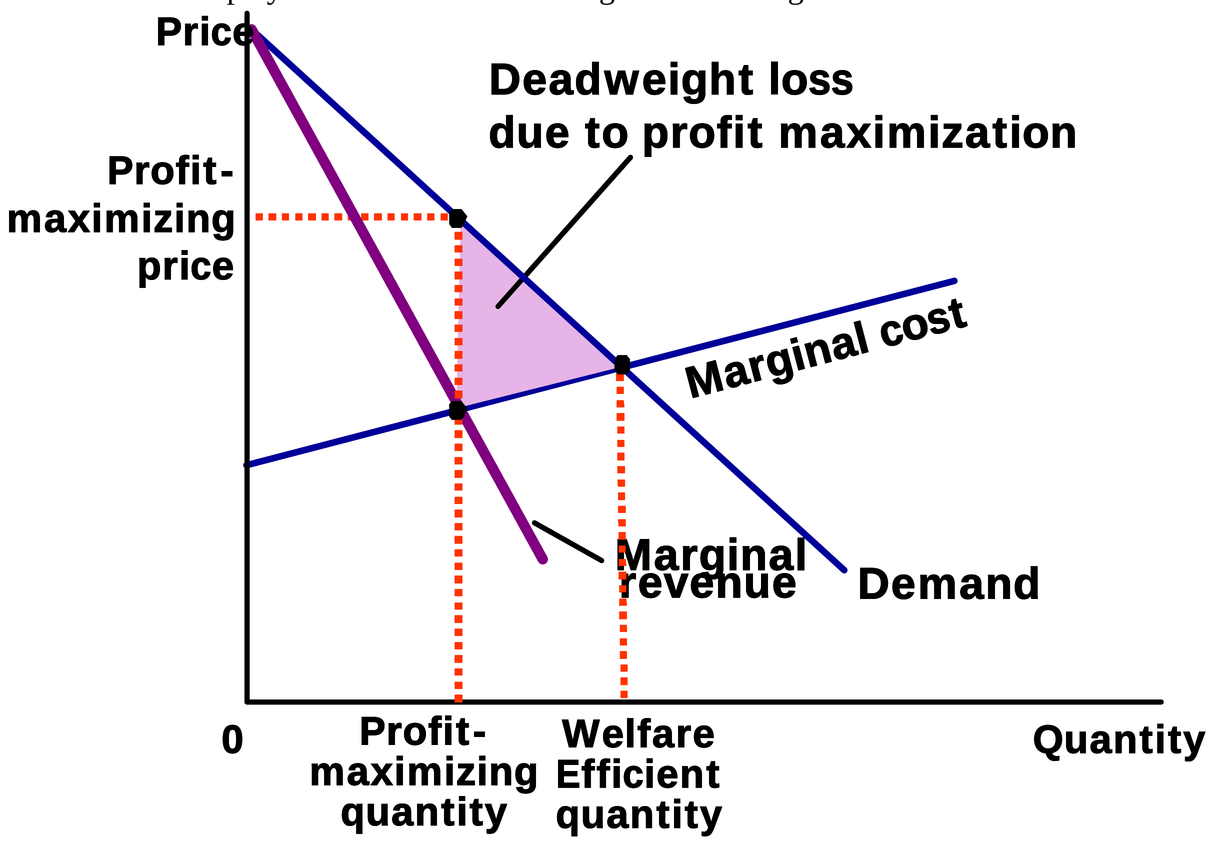

Figure 7: Output and welfare, exceptional welfare comparable case

Therefore in both the special welfare-comparable case of identical constant marginal costs, and the exceptional case of identical operations, competitive firms follow the same profit-maximization rule, a competitive industry produces the same level of output as a monopoly, and the aggregate level of output is generally speaking independent of the number of firms in it.[8] The deadweight loss of welfare that has conventionally been attributed to monopoly behavior is in fact the consequence of profit-maximizing behavior, regardless of the number of firms in the industry. The aggregate market outcome in the exceptional situation is shown in Figure 7, and the correct profit-maximization rule for individual firms in a multi-firm industry is shown in Figure 8.

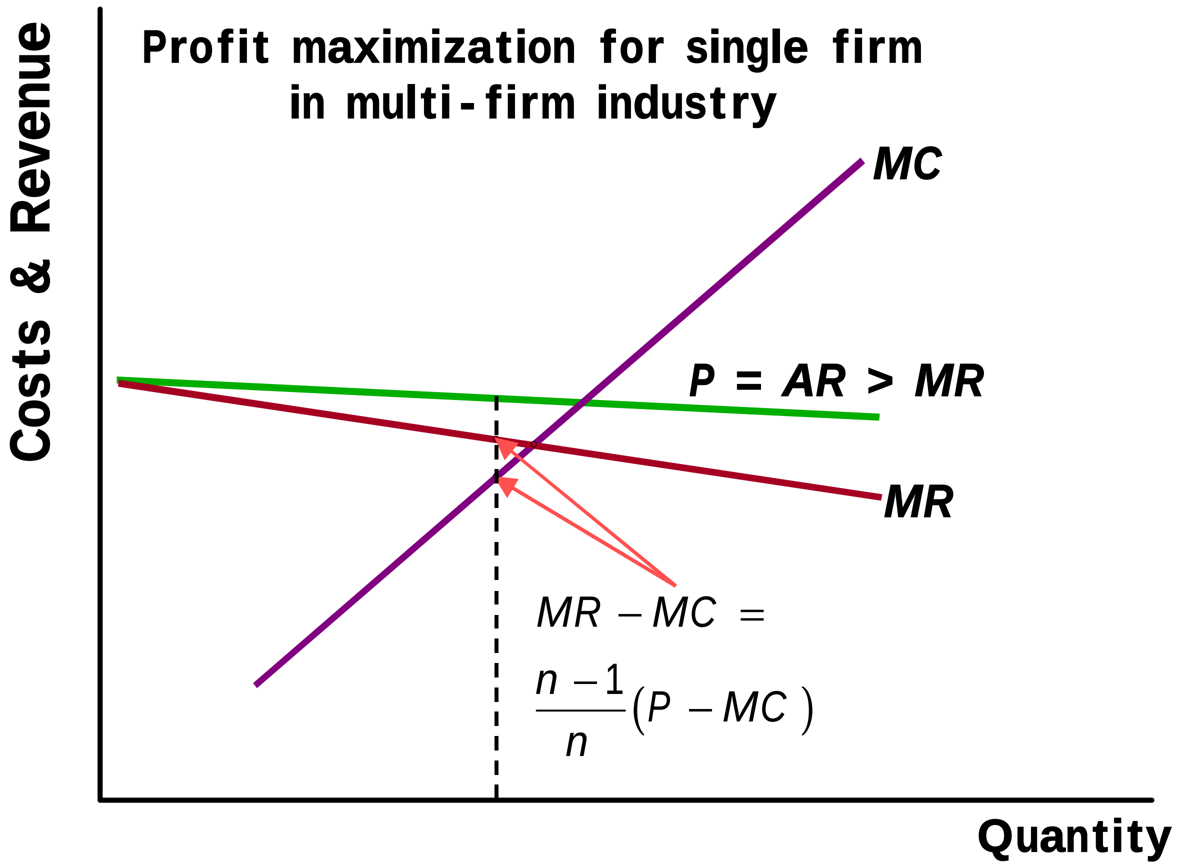

Figure 8: The general profit-maximizing rule in a multi-firm industry

Stigler's relation $MR_i = P + \frac{P}{n \cdot E}$

The preceding analysis facilitates two important observations on Stigler's reworking of marginal revenue as a function of market price, the number of firms, and the market elasticity of demand. Firstly, while Stigler is correct that marginal revenue for the i^{th} firm converges to market price as the number of firms increases, this is exactly offset by the fact that a profit-maximizing firm in an *n*-firm industry sets its own-output marginal revenue *above* marginal cost. Secondly, though convergence of the individual firm's marginal revenue to market price does occur, this is not the so-called competitive price (as Stigler assumed) but the so-called monopoly price.

This can be made explicit by substituting Stigler's Relation for marginal revenue for the i^{th} firm into the correct profit maximization formula and solving for market price:

$$\text{(23)} \quad \begin{aligned} P + \frac{P}{n \cdot E} - MC &= \frac{n-1}{n}(P - MC) \\ \left(1 + \frac{1}{E}\right) \cdot P &= MC \end{aligned}$$

This market price is clearly greater than marginal cost,[9] rather than equal to it as Stigler inferred from the accepted but erroneous profit maximization formula. As is well-known, the LHS of (22) is aggregate industry marginal revenue: $MR = \left(1 + \frac{1}{E}\right) \cdot P$. Price in a competitive industry with profit-maximizing firms therefore converges to the "monopoly" price where aggregate marginal revenue equals aggregate marginal cost, regardless of the number of firms in the industry.

General non-welfare-comparable case: $n \cdot m \equiv x \cdot y, n \gtreqless m, x \gtreqless y, n \cdot f(x) \neq m \cdot g(y)$

The general situation can be inferred from the preceding results. Both the monopoly and the competitive industry structure set quantity so that aggregate marginal cost equals aggregate marginal revenue. Therefore whichever market structure has the lower marginal costs will, according to properly amended theory, produce the greater output.

Whether a competitive industry or a monopoly generates the greater level of consumer welfare is thus an empirical question, to be answered by research into industry costs in each instance. However in general, it could be expected that the larger plants that are likely to be associated with more concentrated industry structures will result in lower marginal costs. Rosput (1993) gives an instructive illustration (in the case of gas delivery) of how greater economies of scale can result in lower marginal costs:

> "Simply stated, the necessary first investment in infrastructure is the construction of the pipeline itself. Thereafter, additional units of throughput can be economically added through the use of horsepower to compress the gas up to a certain point where the losses associated with the compression make the installation of additional pipe more economical than the use of additional horsepower of compression. The loss of energy is, of course, a function of, among other things, the diameter of the pipe. Thus, at the outset, the selection of pipe diameter is a critical ingredient in determining the economics of future expansions of the installed pipe: *the larger the diameter, the more efficient are the future additions of capacity and hence the lower the marginal costs of future units of output.*" (Rosput 1993: 288; emphasis added)[10]

With a lower marginal cost structure, the monopoly will produce a greater quantity than the competitive industry and sell it at a lower price, resulting in a higher level of consumer surplus, as shown in Figure 9. Whereas this was a possibility under accepted theory if the cost difference was sufficiently large, it is a certainty with *any* marginal cost difference in the monopoly's favor when the theory is adjusted to eliminate aggregation errors (compare Figure 9 with Figure 2).

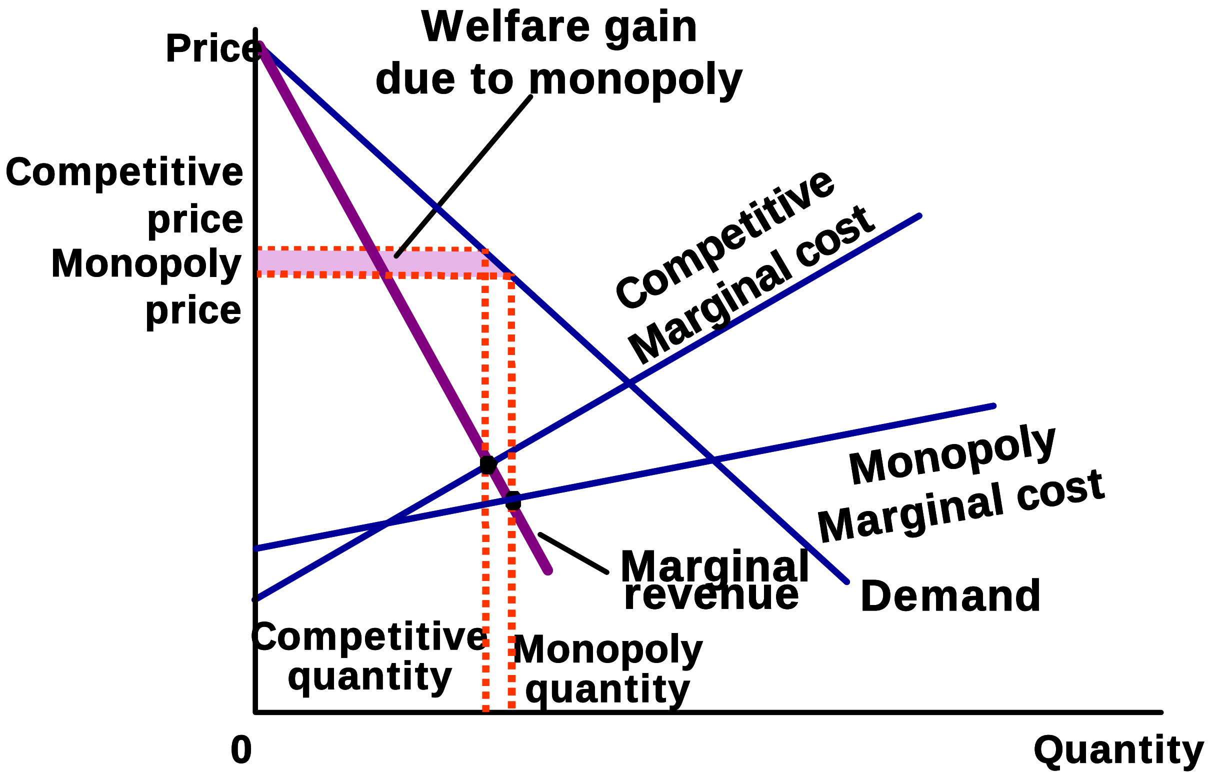

Figure 9: Welfare gain due to monopoly

One final observation on accepted theory can now be made. Our aggregation error-corrected formula shows that aggregate marginal revenue will equal aggregate marginal cost in all industries, so that price will exceed marginal cost in all industries. As is well known, the standard profit-maximization formula results in price converging to marginal cost as the number of firms increases, so that price approximates marginal cost in most industries. The former can be shown to be a true mathematical equilibrium, while the latter is not.

3 Price equals marginal cost not an equilibrium

In general, aggregate profit is

(24) $\Pi(Q) = P(Q) \cdot Q - TC(Q)$

Taking the amended formula first and using Q_K to signify the output level at which aggregate marginal cost equals aggregate marginal revenue, a small change in output ***dq*** results in an aggregate profit level of

(25) $\Pi(Q_K + \delta q) = P(Q_K + \delta q) \cdot (Q_K + \delta q) - TC(Q_K + \delta q)$

Applying a Taylor's series expansion to this, the new profit level is approximately

(26)
$$\Pi(Q_K + \delta q) \approx \left(P(Q_K) + \delta q \cdot \frac{dP}{dQ}\right) \cdot (Q_K + \delta q) - \left(TC(Q_K) + \delta q \cdot \frac{dTC}{dQ}\right)$$
$$= P(Q_K) \cdot Q_K + P(Q_K) \cdot \delta q + \delta q \cdot \frac{dP}{dQ} \cdot Q_K + \delta q \cdot \frac{dP}{dQ} \cdot \delta q - TC(Q_K) - \delta q \cdot \frac{dTC}{dQ}$$

This expansion contains $P(Q_K) \cdot Q_K - TC(Q_K) = \Pi(Q_K)$ which we can now subtract from both sides to yield

(27) $\Pi(Q_K + \delta q) - \Pi(Q_K) \approx \left(P(Q_K) + \frac{dP}{dQ} \cdot Q_K\right) \cdot \delta q + \delta q^2 \cdot \frac{dP}{dQ} - \delta q \cdot \frac{dTC}{dQ}$

Since the output level Q_K is that at which aggregate marginal revenue $P(Q_K) + \frac{dP}{dQ} \cdot Q_K$ equals aggregate marginal cost $\frac{dTC}{dQ}$, the first term on the RHS of (26) ($\left(P(Q_K) + \frac{dP}{dQ} \cdot Q_K\right) \cdot \delta q$) cancels with the third ($\delta q \cdot \frac{dTC}{dQ}$) leaving

(28) $\Pi(Q_K + \delta q) - \Pi(Q_K) \approx \delta q^2 \cdot \frac{dP}{dQ}$

Since $\frac{dP}{dQ} < 0$, this is necessarily negative: hence any change in output will reduce profit.[11] The aggregate output level Q_K is thus a profit-maximizing equilibrium.

Using Q_{PC} for the convergence point of the standard formula, equation (27) becomes

(29) $\Pi(Q_{PC} + \delta q) - \Pi(Q_{PC}) \approx P(Q_{PC}) \cdot \delta q + \delta q \cdot \frac{dP}{dQ} \cdot Q_{PC} + \delta q \cdot \frac{dP}{dQ} \cdot \delta q - \delta q \cdot \frac{dTC}{dQ}$

Since this is the output level at which price equals marginal cost, we have $P(Q_{PC}) = \frac{dTC}{dQ}$. We cancel the first and last terms ($P(Q_{PC}) \cdot \delta q$ and $\delta q \cdot \frac{dTC}{dQ}$) to give us the residual:

(30) $\Pi(Q_{PC} + \delta q) - \Pi(Q_{PC}) \approx \frac{dP}{dQ} \cdot (Q_{PC} + \delta q) \cdot \delta q$

$\frac{dP}{dQ}$ is negative and $(Q_{PC} + \delta q)$ is positive, so the sign of $\Pi(Q_{PC} + \delta q) - \Pi(Q_{PC})$ is the inverse of the sign of ***dq***. If ***dq***>0, profit will fall; if ***dq***<0, profit will rise. Thus any firm that decreases its output from the level at which its marginal cost equals marginal price will increase its profits, and the increase in profit will also to some extent be transmitted to all other firms via an increase in market price. The output level at which price equals marginal cost is therefore mathematically not a profit-maximizing equilibrium.

4 Conclusion

Profit-maximizing firms do not equate their own-output marginal revenue and marginal cost, but instead produce an output at which their own-output marginal revenue exceeds marginal cost. Competitive industries produce an output at which aggregate marginal revenue equals industry marginal cost, regardless of the number of firms in the industry. Definitive welfare comparisons can only be made in the two special cases of constant identical marginal costs or monopolization by takeover. In all other cases a monopoly will produce a higher output at a lower cost than a competitive industry if—as is likely given economies of scale—it operates with a lower marginal cost. Therefore according to the aggregation error amended theory of the firm, price will be greater than marginal cost in a competitive profit-maximizing equilibrium, and consumer welfare can be expected to be higher under monopoly than under competition.

These results constitute major revisions to accepted neoclassical analysis, with serious implications from undergraduate instruction at one extreme to welfare analysis, general equilibrium theory and the field of Industrial Organization at the other. In the policy field they reverse the conventional neoclassical preference for competitive firms over monopolies and other less competitive structures.

There are two possible courses of action in the light of our results. One is to try to find other methods by which the traditional results can be reached. The other is to develop an alternative theory of the firm.

The former could perhaps be undertaken using game theory,[12] in which perfect competition is the limit of the Prisoners' Dilemma "defect" strategy as the number of firms rises indefinitely. However there are several problems with this approach.

Firstly, it is well-known that the iterated Prisoners' Dilemma does not necessarily converge to the defection solution (Richard Schmalensee, 1988: 647).

Secondly, there are serious dimensionality issues in extending the analysis of single period *2x2* player/strategy combinations to those with multiple periods, *n* players, and multiple feasible coalitions and strategies (Robert Marks, 2000). In a single period 2 person 2 strategy game there are 4 possible outcomes. Leaving aside the issue of coalitions, the number of potential states scales as $Strategies^{Players \cdot Rounds}$, so that a 50 person 3 period game has over 10^{33} possible states. It is difficult to sustain that competitive firms could be strategically analyzing this or a greater number of outcomes in making their output decisions.

Thirdly, no alternative route can circumvent the marginal cost aggregation problem, so that any analytic conclusions (such as the welfare-superiority of competition over monopoly) would remain tentative without empirical evidence on the cost functions of competitive and monopoly firms.

This leaves the second course of developing an alternative theory of the firm.

Many possible routes could be taken here; we will suggest only one, that this new theory should attempt to explain the observed behavior of actual firms. Numerous empirical studies (Wilford J. Eiteman, 1947, Walter W. Haines, 1948, Gardiner C. Means, 1972, Alan S. Blinder et al., 1998; see Frederic S. Lee, 1998 and Paul Downward and Frederic S. Lee, 2001 for surveys) have shown that the vast majority of actual firms have production functions characterized by large average fixed costs and constant or falling marginal costs. This cost structure, which is described as "natural monopoly" in economic literature and portrayed as an exception to the rule of rising marginal cost, actually appears to be the empirical reality for between 89 per cent and 95 per cent of firms and products (Downward & Lee 2001: 469, citing Blinder et al. 1998; Eiteman & Guthrie 1952: 837). Though hypothetically marginal cost could exceed average cost at above capacity levels of output, in practice marginal cost lies well below average cost because firms install and maintain substantial excess capacity in order to cope with expanding demand over time and uncertain marketing conditions.[13]

In general, these studies conclude that firms determine their prices by a markup on variable costs, with the size of the markup reflecting partly the need to cover fixed costs at a levels of output well within production capacity, the desire to finance investment and/or repay debt with retained earnings, the impact of the trade cycle, and the degree of competition (so that empirical research gives some grounds by which a more competitive industry can be preferred to a less competitive one). Price is set by the firm prior to the market, and the firm attempts to sell as much of its output as it can at this price. Firms produce competing but heterogeneous products, and the main form of competition between firms is not price but product differentiation (by both marketing and R&D). Once a breakeven sales level has been achieved, each new unit sold adds significantly to profit, and this continues out to the last unit sold—so that marginal revenue is always significantly above marginal and average cost. Output is constrained not by increasing production costs as output rises, but by the difficulty of expanding sales in a heterogeneous market environment, and the financial risks involved in investing in larger production facilities.

Means coined the term "the administered price thesis" for this perspective on pricing behaviour (Means 1972), while the vision of competitive behaviour is consonant with the dynamic theory of competition and creative destruction proposed by Schumpeter (1936).

A major theme in Friedman's influential paper on methodology (Milton Friedman, 1953) was that this empirical literature could be ignored. Starting with the example that billiard players did not explicitly apply Newton's laws when shooting billiards, but that if they did not behave *as if* they applied these laws then "they would not in fact be *expert* billiard players." (Friedman 1953: 21), Friedman continued that

> It is only a short step from these examples to the economic hypothesis that ... individual firms behave as if ... they ... calculated marginal cost and marginal revenue from all actions open to them, and pushed each line of action to the point at which the relevant marginal cost and marginal revenue were equal. ... The billiard player ... may say that he "just figures it out" ...; and the businessman may well say that he prices at average cost, with of course some minor deviations when the market makes it necessary. The one statement is about as helpful as the other, and neither is a relevant test of the associated hypothesis. (Friedman 1953: 22)

Friedman's recommendation was based on confidence that the accepted neoclassical profit maximization formula was intellectually sound (see also Lawrence A. Boland, 1981). Our research shows that this is not so. In the light of this, we submit that this empirical literature should be reconsidered by theoretical and practical microeconomists, and made the foundation for a new microeconomics of the firm.

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2 As a result, marginal revenue equals price for competitive firms, so that profit maximizing behaviour—equating marginal cost to marginal revenue—results in competitive firms individually and collectively setting price equal to marginal cost.

3 dQ_R here is simply changes that all other firms are making to output as they independently search for a profit-maximizing level of output, and not changes in the output of the rest of the industry in direct response to a change in output by the i^{th} firm, which is zero for independent firms.

4 This has nothing to do with fixed costs, which are not being explicitly considered at this point and can be regarded as zero.

5 The summation term is positive since though aggregate industry marginal revenue is much less than price, own output marginal revenue is greater than (n-1)/n times market price.

6 The case of non-zero initial marginal cost can be derived by combining the results of this section with the previous one.

7 That is, the competitive industry has *n* firms employing *x* workers each, and the monopoly operates *n* plants with *x* workers, each with the same production functions as the competitive firms

8 With the exception of the effect of dividing output between multiple plants in the exceptional welfare comparison case, where the competitive industry and the monopoly still produce the same level of output.

9 Since $E = \frac{P}{Q} \cdot \frac{dQ}{dP} < 0$.

10 Economies of scale in research and technology will also favor larger-scale operations. While these are ruled out in the standard comparison with identical marginal cost curves, they cannot be ruled out in the general case of differing marginal cost curves.

11 This will apply to any individual firm that changes its output; though of course the impact of its change will be felt by all firms in the industry.

12 Contestability obviously can't resuscitate the conventional results, since our profit maximization results apply across all other industries.

13 Such as the safety recall of Firestone tyres in 2000 that proved such a boon to other tyre manufacturers.